1. Distortion in Nonlinear Systems

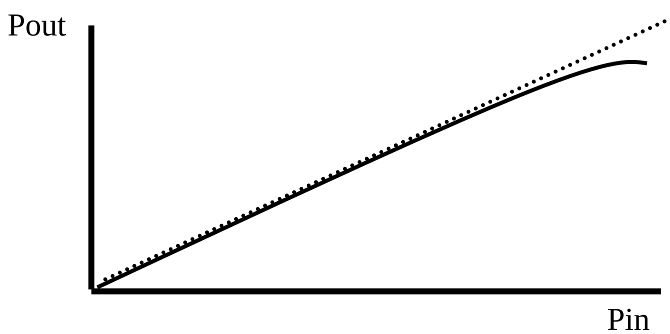
The upper limit of useful operation is limited by distortion. All analog systems and components of systems (amplifiers and mixers for example) become nonlinear when driven at large signal levels. The nonlinearity distorts the desired signal. This distortion exhibits itself in several ways:

1. Gain compression or expansion (sometimes called AM – AM distortion)
2. Phase distortion (sometimes called AM – PM distortion)
3. Unwanted frequencies (spurious outputs or spurs) in the output spectrum. For a single input, this appears at harmonic frequencies, creating *harmonic distortion* or HD. With multiple input signals, in-band distortion is created, called *intermodulation distortion* or IMD.

When these spurs interfere with the desired signal, the S/N ratio or SINAD (Signal to noise plus distortion ratio) is degraded.

Gain Compression.

The nonlinear transfer characteristic of the component shows up in the grossest sense when the gain is no longer constant with input power. That is, if Pout is no longer linearly related to Pin, then the device is clearly nonlinear and distortion can be expected.



P_{1dB}, the input power required to compress the gain by 1 dB, is often used as a simple to measure index of gain compression. An amplifier with 1 dB of gain compression will generate severe distortion.

Distortion generation in amplifiers can be understood by modeling the amplifier's transfer characteristic with a simple power series function:

$$V_{out} = a_1 V_{in} - a_3 V_{in}^3$$

Of course, in a real amplifier, there may be terms of all orders present, but this simple cubic nonlinearity is easy to visualize. The coefficient a_1 represents the linear gain; a_3 the distortion. When the input is small, the cubic term can be very small. At high input levels, much nonlinearity is present. This leads to gain compression among other undesirable things. Suppose an input Vin =A sin (ωt) is applied to the input.

$$V_{out} = A\left[a_1 - \frac{3a_3A^2}{4}\right]\sin(\omega t) + \frac{1}{4}a_3A^3\sin(3\omega t)$$

Gain Compression Third Order Distortion

Gain compression is a useful index of distortion generation. It is specified in terms of an input power level (or peak voltage) at which the small signal conversion gain drops off by 1 dB.

The example above assumes that a simple cubic function represents the nonlinearity of the signal path. When we substitute $V_{in}(t)$ = A sin (ωt) and use trig identities, we see a term that will produce gain compression:

$A(a_1 - 3a_3A^2/4)$.

If we knew the coefficient a_3, we could predict the 1 dB compression input voltage. Typically, we obtain this by measurement of gain vs. input voltage.

Harmonic Distortion

We also see a cubic term that represents the third-order *harmonic distortion* (HD) that also is caused by the nonlinearity of the signal path. Harmonic distortion is easily removed by filtering; it is the *intermodulation distortion* that results from multiple signals that is far more troublesome to deal with.

Note that in this simple example, the fundamental is proportional to A whereas the third-order HD is proportional to A^3. Thus, if Pout vs. Pin were plotted on a dBm scale, the HD power will increase at 3 times the rate that the fundamental power increases with input power. This is often referred to as being "*well behaved*", although given the choice, we could easily live without this kind of behavior!

Intermodulation Distortion

Let's consider again the simple cubic nonlinearity $a_3 v_{in}^3$. When two inputs at ω_1 and ω_2 are applied simultaneously to the RF input of the system, the cubing produces many terms, some at the harmonics and some at the IMD frequency pairs. The trig identities show us the origin of these nonidealities. [4]

Let's consider the 3rd order nonlinearity: $a_3 v_{in}^3$

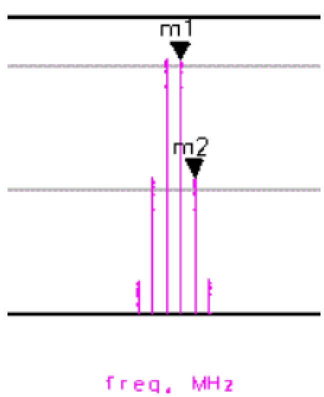
– two inputs: $v_{in} = V_1 \sin(\omega_1 t) + V_2 \sin(\omega_2 t)$

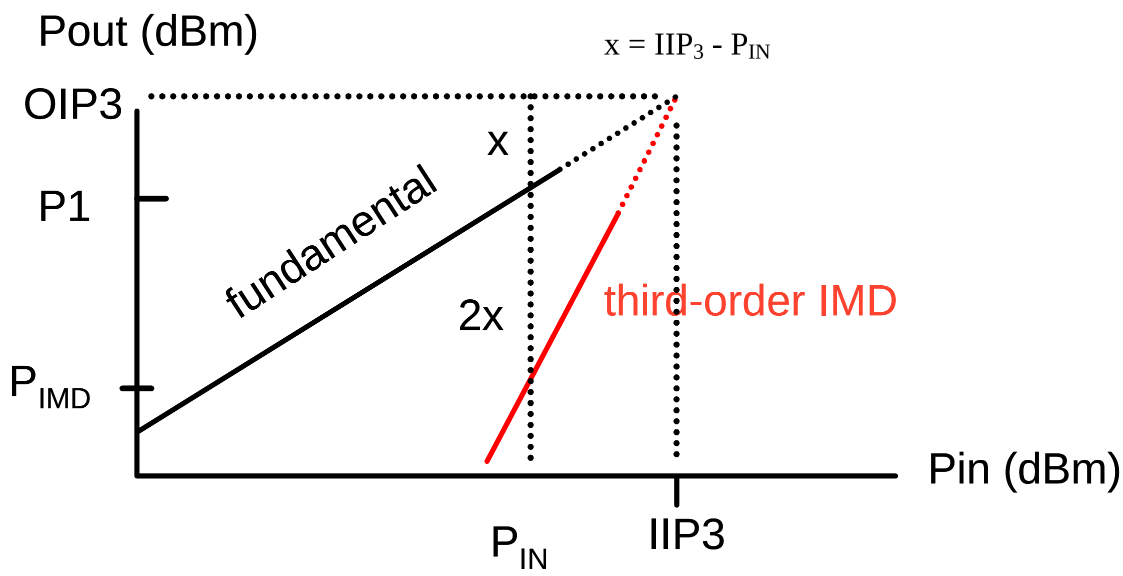
$$V_{out3} = a_3[V_1^3 \sin^3(\omega_1 t) + V_2^3 \sin^3(\omega_2 t) + 3V_1^2 V_2 \sin^2(\omega_1 t)\sin(\omega_2 t) + 3V_1 V_2^2 \sin(\omega_1 t)\sin^2(\omega_2 t)]$$

$$\frac{3V_1^2 V_2 a_3}{2}\left\{\sin(\omega_2 t) - \tfrac{1}{2}[\sin(2\omega_1 - \omega_2)t - \sin(2\omega_1 + \omega_2)t]\right\}$$

Cross-modulation Third-order IMD

We will be mainly concerned with the third-order IMD. (actually, any distortion terms can create in-band signals – we will discuss this later). IMD is especially troublesome since it can occur at frequencies within the signal bandwidth. For example, suppose we have 2 input frequencies at 899.990 and 900.010 MHz. Third order products at $2f_1 - f_2$ and $2f_2 - f_1$ will be generated at 899.980 and 900.020 MHz. These IM products may fall within the filter bandwidth of the system and thus cause interference to a desired signal. The spectrum would look like this, where you can see both third and fifth order IM.





$$IIP_3 = P_{IN} + \frac{1}{2}\left(P_1 - P_{IMD}\right)$$

IMD power, just as HD power, will have a slope of 3 on a Pout vs Pin (dBm) plot. A widely-used figure of merit for IMD is the *third-order intercept* (TOI) point. This is a fictitious signal level at which the fundamental and third-order product terms would intersect. In reality, the intercept power is 10 to 15 dBm higher than the P_{1dB} gain compression power, so the circuit does not amplify or operate correctly at the IIP_3 input level. The higher the TOI, the better the large signal capability of the system. If specified in terms of input power, the intercept is called IIP_3. Or, at the output, OIP_3. This power level can't be actually reached in any practical amplifier, but it is a calculated figure of merit for the large-signal handling capability of any RF system.

It is common practice to extrapolate or calculate the intercept point from data taken at least 10 dBm below P_{1dB}. One should check the slopes to verify that the data obeys the expected slope = 1 or slope = 3 behavior. The TOI can be calculated from the following geometric relationship:

$$OIP_3 = (P_1 - P_{IMD})/2 + P_1$$

Also, the input and output intercepts (in dBm) are simply related by the gain (in dB):

$$OIP_3 = IIP_3 + \text{power gain.}$$

Other higher odd-order IMD products, such as 5th and 7th, are also of interest, and can also be defined in a similar way, but may be less reliably predicted in simulations unless the device model is precise enough to give accurate nonlinearity in the transfer characteristics up to the $2n\text{-}1^{th}$ order.

Cross Modulation

In addition, the cross-modulation effect can also be seen in the equation above. The amplitude of one signal (say ω_1) influences the amplitude of the desired signal at ω_2 through the coefficient $3V_1^2V_2a_3/2$. A slowly varying modulation envelope on V_1 will cause the envelope of the desired signal output at ω_2 to vary as well since this fundamental term created by the cubic nonlinearity will add to the linear fundamental term. This cross-modulation can have annoying or error generating effects at the output.

Second Order Nonlinearity

In the simplified model above, we have neglected second order nonlinear terms in the series expansion. In many cases, an amplifier or other RF system will have some even-order distortion as well. The transfer function then would look like this:

$$V_{out} = a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3$$

If we once again apply two signals at frequencies ω_1 and ω_2 to the input, we obtain:

$$V_{out2} = a_2\left[V_1^2 \sin^2(\omega_1 t) + V_2^2 \sin^2(\omega_2 t) + 2V_1V_2 \sin(\omega_1 t)\sin(\omega_2 t)\right]$$

The $\sin^2$ terms expand into:

$$\frac{1}{2}a_2V_1^2\left[1-\cos(2\omega_1 t)\right] + \frac{1}{2}a_2V_2^2\left[1-\cos(2\omega_2 t)\right]$$

From this, we can see that there is a DC term and a second harmonic term present for each input. The DC term is proportional to the square of the voltage, therefore power. This is one use of second-order nonlinearity – as a power sensor. The HD term is also proportional to the square of the voltage. Thus, on a power out vs. power in plot, it has a slope of 2.

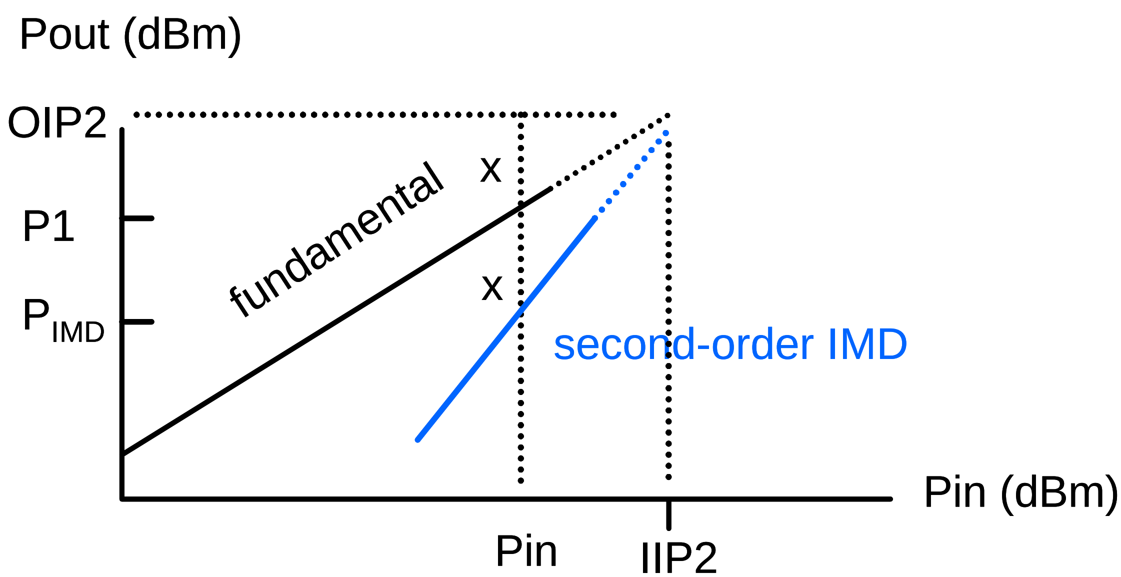
When the next term is expanded, the product of two sine waves is seen to produce the sum and difference frequencies.

$$a_2 V_1 V_2 \left[\cos(\omega_2 - \omega_1)t - \cos(\omega_2 + \omega_1)t\right]$$

This can be both a useful property and a problem. The useful application is as a frequency translation device, often called a mixer, a downconverter, or an upconverter. The desired output is selected by inserting a filter at the output of the device.

Second order distortion, if generated by out-of-band signals, can also lead to interference in-band as shown below. Preselection filtering can generally suppress this in narrowband amplifiers, but it can be a big problem for wideband circuits.

A SOI, or second-order intercept can also be defined as shown below:

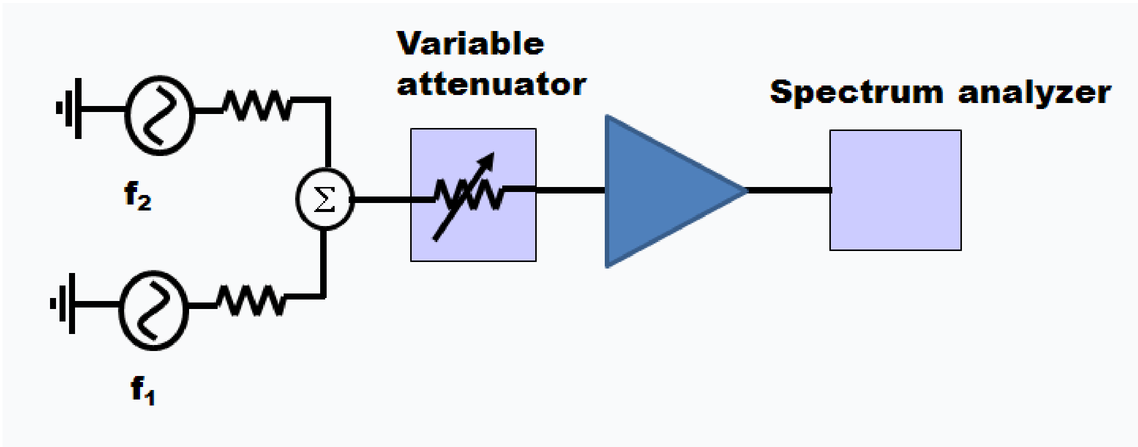


The second-order IMD slope = 2. IIP_2 can be calculated from measurement by:

$$IIP_2 = P_{in} + P_1 - P_{IMD}$$

$$OIP_2 = IIP_2 + \text{Power Gain} = 2P_1 - P_{IMD}$$

Measuring Intermodulation Distortion



Set the amplitude of generators at f1 and f2 to be equal.

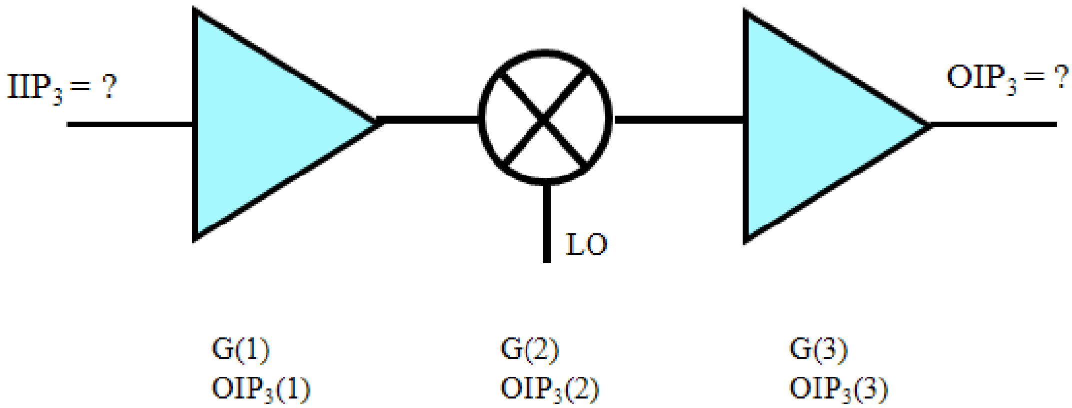
Start at a very low input power using the variable attenuator, then increase power in steps until you begin to see the IMD output on the spectrum analyzer. The resolution bandwidth should be narrow so that the noise floor is reduced. This will allow visibility of the IMD signal at lower power levels.

Plot the IMD power vs. input power and verify that the slope is close to 3. Then, you can calculate the IIP_3 as described previously.

Two tone simulation in ADS

Refer to the first part of the Harmonic Balance Simulation Tutorial on the course web page.

How is the Third-Order Intercept Point affected by cascaded stages?



Gains multiply in a cascade: $P_O = P_i\ G(1)\ G(2)\ G(3)$ (or add them if in dB)

Individual intercept points must be referred to the same reference plane. It can be either at the input or the output. In this example, the output TOI, OIP_3, is specified for each stage.

1. Convert all OIPs from dBm to mW and gains from dB to a power ratio.
2. Let's refer all of these OIPs to the output plane.

   OIP_3

   $G(3)\ OIP_3(2)$

   $G2)\ G(3)\ OIP_3(1)$

3. The **third order intercept** cascading relationship is:

$$\frac{1}{OIP_3} = \frac{1}{G(2)G(3)OIP_3(1)} + \frac{1}{G(3)\,OIP_3(2)} + \frac{1}{OIP_3(3)}$$

$$IIP_3 = \frac{OIP_3}{G(1)G(2)G(3)}$$

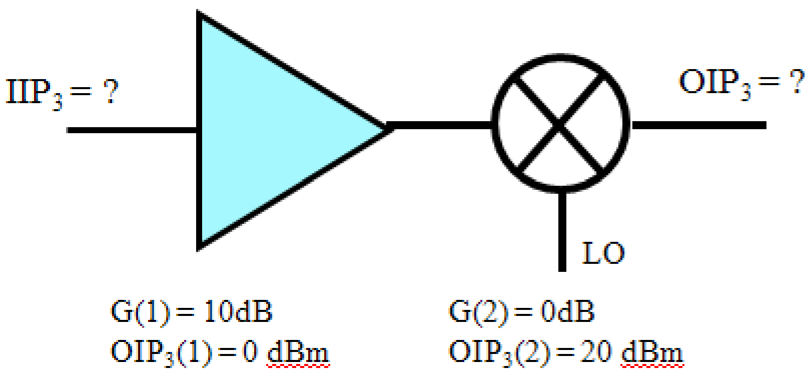
4. Convert the results back to dBm if desired.

Second order intercept cascading is accomplished by the following equations:

$$\frac{1}{\sqrt{OIP_2}} = \frac{1}{\sqrt{G(2)G(3)OIP_2(1)}} + \frac{1}{\sqrt{G(3)\,OIP_2(2)}} + \frac{1}{\sqrt{OIP_2(3)}}$$

$$IIP_2 = \frac{OIP_2}{G(1)G(2)\,G(3)}$$

Example: Third-order intercept of a receiver front end



1. Convert dBm to mW: $OIP_3(1)$ = 1 mW, $OIP_3(2)$ = 100 mW

   Convert dB to a power ratio: G(1) = 10, G(2) = 1
2. Refer to the output plane:

   $1/OIP_3 = 1 + 1/100 = 1.01$ $OIP_3 = 1$ (0 dBm)
3. $IIP_3 = OIP_3/10 = 0.1$ (-10 dBm)

We can see that the LNA completely dominates the IIP_3 in this example. IF we eliminated the LNA, then $OIP_3 = OIP_3(2)$ = 20 dBm and IIP = 20 dBm, **a 30 dB improvement!**

What do we lose by eliminating the LNA?

From W. Hayward, Intro. to RF Design, ARRL Press, 1994.

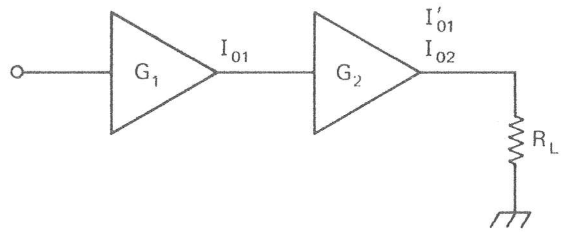


Figure 6.18 A cascade of two amplifiers, each with a known output intercept. I'_{01} is the output intercept of the first stage renormalized to the output plane, achieved by increasing I_{01} by G_2, the second stage gain. If the distortion products are assumed coherent, and all intercepts are normalized to one plane, the equivalent intercept is calculated just as the net resistance of parallel resistors is evaluated.

Consider now the more general case where both amplifiers have finite output intercepts. The analysis will be confined to third order imd although the approach is easily extended to distortion of any order. Assume that the intercepts of both stages have been normalized to the same plane in the cascade. The intercepts will be designated by I_n where the subscript n denotes the stage. D_n will refer to a distortion power while P_n will describe the desired output power of the nth stage normalized to the plane of interest.

If the fundamental defining concepts of the intercept are invoked in algebraic terms instead of logarithmic units, the distortion power of the nth stage is $D_n = P_n^3/I_n^2$. This power appears in a load resistance, R. Hence, the corresponding distortion voltage is $V_n = (RD_n)^{1/2} = (P^3R)^{1/2}/I_n$. The total distortion will come from the addition of the distortion voltages.

As was the case with noise voltages, distortion voltages must be added with care. If the voltages are phase related, they should be added algebraically. However, if they are completely uncorrelated, they will add just as thermal noise voltages do, as the root of the sum of the squares. There is usually a well-defined phase relationship between signals with amplifiers. The worst case is when distortions from two stages add exactly in phase. This will lead to the largest distortion. Some cases may exist where distortion voltages are coherent (phase related) and cancel to lead to a distortionless amplifier. Like most physical phenomena, this is unusual and not the sort of thing that a designer can depend upon. We will take the conservative approach of choosing the worst possible case, that of algebraic addition of the distortion voltage, assuming them to be in phase.

Using the worst case assumption, the total distortion voltage is $V_T = V_1 + V_2$

$$V_T = \left(\frac{1}{I_1} + \frac{1}{I_2}\right)(P^3R)^{1/2} \tag{6.3-10}$$

The corresponding power is then

$$D_T = \frac{V_T^2}{R} = P^3\left(\frac{1}{I_1} + \frac{1}{I_2}\right)^2 \tag{6.3-11}$$

From the earlier definition, the net or total intercept at the plane of definition is

$$I_T = (P^3/D_T)^{1/2} \tag{6.3-12}$$

Further manipulation yields the final result

$$I_T = \left(\frac{1}{I_1} + \frac{1}{I_2}\right)^{-1} \tag{6.3-13}$$

Equation 6.3-13 has a familiar form with an easy to remember analogy. If intercepts are normalized to a single plane and are expressed as powers in milliwatts or watts rather than logarithmic units, the total intercept at the plane of definition is a sum similar to that for resistors in parallel. This applies only for the case of coherent addition of distortion voltages for third order imd. Not only is this analysis conservative to the extent that it is "worst case," but it works well in practice, predicting measured results with reasonable accuracy.

Consider an example, two identical amplifiers with a gain of 10 dB and an output intercept of +15 dBm. If the two intercepts are normalized to the corresponding ones at the output, they are +15 and +25 dBm. Converting to milliwatts, the two intercepts are 31.62 and 316.2. Application of the resistors-in-parallel rule yields an equivalent output intercept of 28.75 mW, or 14.59 dBm. Essentially, the imd is completely dominated by the output stage.

A more realistic design would be one with a "stronger" second stage. Assume that the output intercept of the second stage is increased to +25 dBm. That of the first stage is still +15 dBm, while both gains remain at 10 dB. The result is an ouput intercept of +22 dBm. The output intercept of the first stage equals the input intercept of the second to yield equal distortion contribution from each and a 3-dB degradation over the intercept of an individual stage.

Generally, the last stage in a chain will determine the third order imd performance. This will be maintained so long as the output intercept of the previous stage is greater than the input intercept of the last.

Some generalizations may be made about the intercepts of some amplifiers. Consider first the question of gain compression in a common emitter bipolar amplifier. From an intuitive viewpoint, we would expect the gain to begin to decrease significantly when the collector signal current reaches a peak value equaling the dc bias current. The signal current will then be varying from the bias level to twice that value and to zero on negative-going peaks. This assumes that the supply voltage is high enough that no voltage limiting occurs. The load also effects the possibility of voltage limiting.

It is found experimentally that the 1-dB gain compression point is well approximated by the current limiting described. Gain will still be present at higher levels and the continued gain compression is gradual until a "saturated" output is reached. Distortion is severe at high levels above the point of 1-dB compression. A bipolar transistor with a 50-Ω collector termination will have a 1-dB compression point of

2. Next Topic: NOISE

Noise determines the minimum signal power (minimum detectable signal or MDS) at the input of the system required to obtain a signal to noise ratio of 1. A S/N = 1 is usually considered to be the lower acceptable limit except in systems where signal averaging or processing gain is used. Noise figure is a figure of merit used to describe the amount of degradation in S/N ratio that the system introduces as the signal passes through.

For some applications, the minimum signal power that is detectable is important.

- Satellite receiver
- Terrestrial microwave links
- 802.11

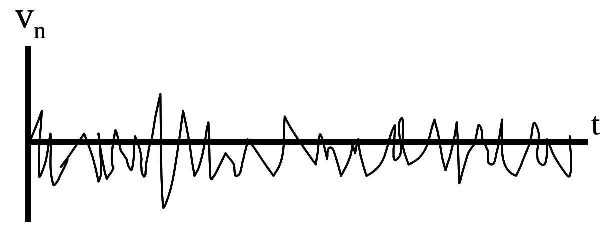
Noise limits the minimum signal that can be detected for a given signal input power from the source or antenna.

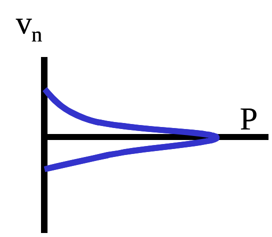
We will identify sources of noise, and define related quantities of interest:

- S/N = Signal to noise ratio
- MDS = Minimum Detectable Signal
- F = Noise factor
- NF = 10 * log(F) = Noise figure

Noise Basics:

What is noise? How is it evident to us? Why is it important?





What:

1. Any unwanted random disturbance
2. Random carrier motion produces a current. Frequency and phase are not predictable at any instant in time
3. The noise amplitude is often represented by a Gaussian probability density function.

The cumulative area under the curve represents the probability of the event occurring. Total area is normalized to 1.

Because of the random process, the average value is zero:

$$\bar{v}_n = \lim_{T \to \infty} \frac{1}{T} \int_{t_1}^{t_1+T} [v_n(t)] \, dt = 0$$

We cannot predict $v_n(t)$, but the variance (standard deviation) is finite:

$$\overline{v_n^2} = \lim_{T\to\infty} \frac{1}{T} \int_{t_1}^{t_1+T} [v_n(t)]^2 \, dt = \sigma^2$$

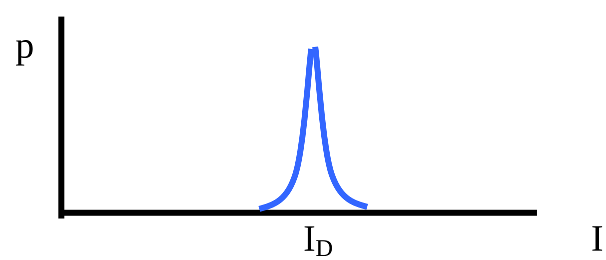
Often we refer to the rms value of the noise voltage or current:

$$v_{n,rms} = \sqrt{\overline{v_n^2}}$$

Sources of Noise in Circuits:

- Shot noise — forward-biased junctions
- Thermal Noise — any resistor
- Flicker (1/f) noise — trapping effects

Shot noise: This is due to the random carrier flow across a pn junction. Electrons and holes randomly diffuse across the junction producing noise current pulses that occur randomly in time. The steady state current measured across a forward biased diode junction is really a large number of discrete current pulses.



The variance of this current:

$$\overline{i^2} = \lim_{T\to\infty} \frac{1}{T} \int_0^T (I - I_D)^2 \, dt = \sigma^2$$

It can be shown that this mean square noise current can be predicted by

$$\overline{i^2} = 2qI_D B$$

where

q = charge of an electron = 1.6 x 10^{-19}

I_D = diode current

B = bandwidth in Hertz (sometimes called Δf)

The noise current spectral density: $\overline{i^2}/B = 2qI_D$

- Independent of frequency (white noise)
- Independent of temperature for a fixed current
- Proportional to the forward bias current
- Gaussian probability distribution

1 mA of current corresponds to a noise current spectral density of

18 pA/√Hz

read: 18 picoamp per root Hertz

Thermal Noise: Thermal noise, sometimes called Johnson noise, is due to random motion of electrons in conductors. It is unaffected by DC current and exists in all conductors. Its spectral density is also frequency independent, but is directly proportional to temperature. The noise probability density is Gaussian.

$$\overline{v^2} = 4kTRB$$

$$\overline{i^2} = 4kTB/R$$

4kT = 1.66 x 10^{-20} V-C

A 50 ohm resistor produces a noise voltage spectral density of

0.9 nV/√Hz

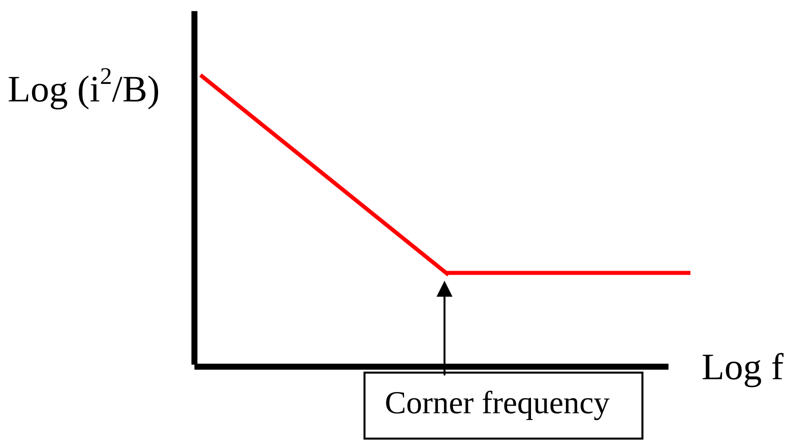
or a Norton equivalent noise current spectral density of

18 pA/√Hz

Flicker or 1/f noise. This noise source is most evident at very low frequencies. It is hard to localize its physical mechanisms in most devices. There is usually some 1/f noise contribution due to charge traps with long time constants. The trap charge then is randomly released after some relatively long period of time. 1/f noise is modeled by:

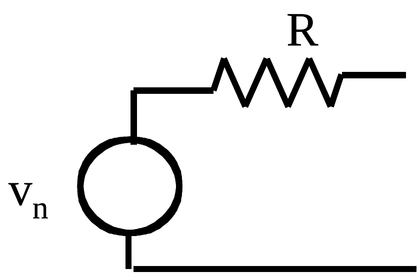
$$\bar{i}^2 / B = K \frac{I}{f}$$

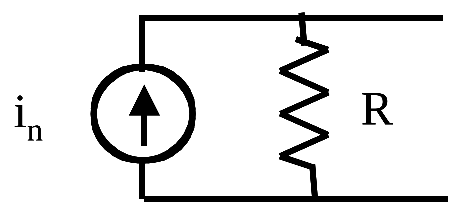
- ❖ K is a fudge factor. It can vary wildly from one type of transistor to the next or even from one fabrication lot to the next.
- ❖ I is the current flowing through the device.
- ❖ B is the bandwidth.



- ❖ 1/f noise can be described by a corner frequency.
- ❖ Carbon resistors exhibit 1/f noise; metal film resistors do not.

Noise can be modeled as a Thevenin equivalent voltage source or a Norton equivalent current source. The noise contributed by the resistor is modeled by the source, thus the resistor is considered noiseless.

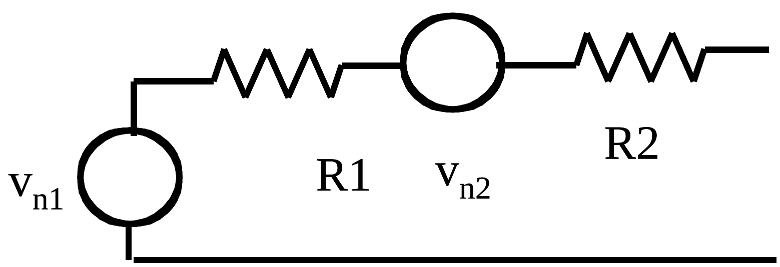




It is important to note that noise sources:

- Do not have polarity (the arrow is just to distinguish current from voltage)
- Do not add algebraically, but as RMS sums

$$\overline{v}^{2}_{n,total} = \overline{v}^{2}_{n1} + \overline{v}^{2}_{n2} = 4kTBR_1 + 4kTBR_2$$



If the sources are correlated (derived from the same physical noise source), then there is an additional term:

$$\overline{v}^{2}_{n,total} = \overline{v}^{2}_{n1} + \overline{v}^{2}_{n2} + 2C v_{n1} v_{n2}$$

C can vary between –1 and 1.

The <u>available noise power</u> can be calculated from the RMS noise voltage or current:

$$P_{av} = \frac{\overline{v_n^2}}{4R} = \frac{\overline{i_n^2}R}{4} = kTB$$

That is, the available noise power from the source is

- independent of resistance
- proportional to temperature
- proportional to bandwidth
- has no frequency dependence

$$P_{av} = 4 \times 10^{-21} \text{ watt}$$

in a 1 Hz bandwidth at the standard noise room temperature of 290 K. If converted to dBm = 10 log(P/10^{-3}), this power becomes

- 174 dBm/Hz

We are generally interested in the noise power in other bandwidths than 1 Hz. It's easy to calculate: P = kTB where kT = -174 dBm

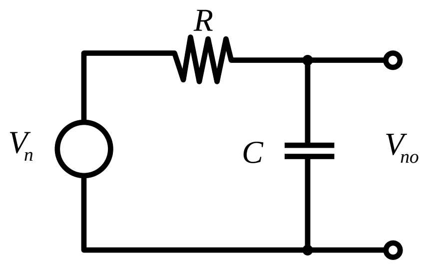
To convert bandwidth in Hertz to dB: 10 log B

EX: Suppose your B = 1000 Hz. P = kTB.

In dBm, P = -174 + 10 log (1000) = -174 + 30 = -144 dBm

Can a resistor produce infinite noise voltage?

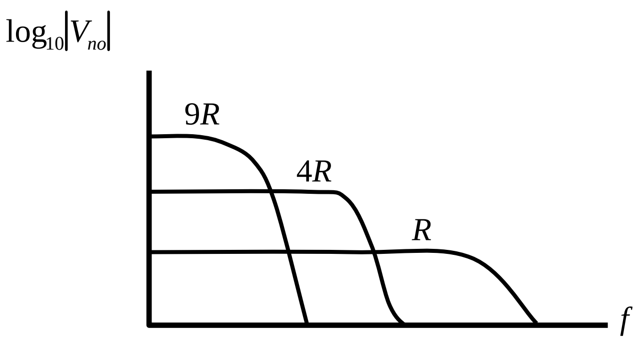
$$V_n^2 = 4kTBR$$



Equivalent circuit for noisy resistor.

Always some shunt capacitance.

Low Pass



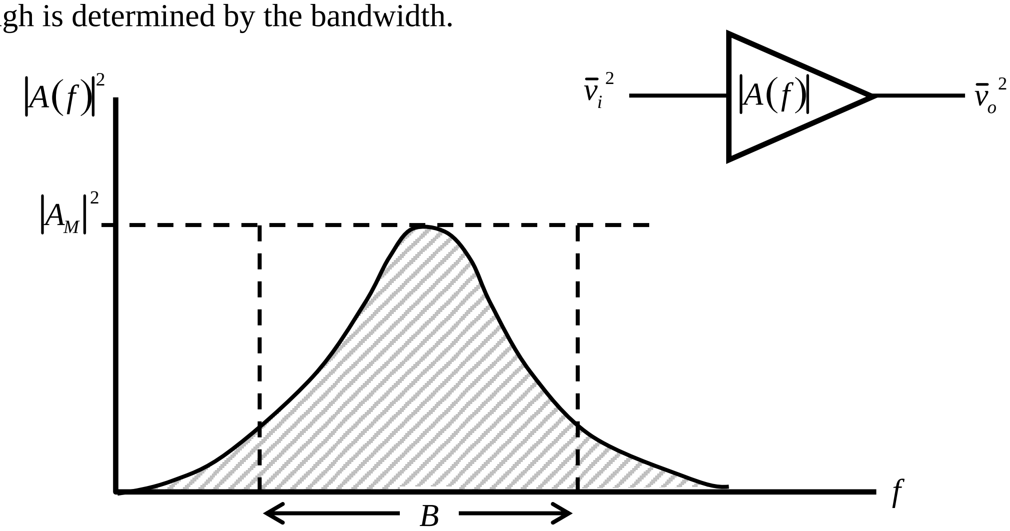
$$|V_{no}| = |V_n| \frac{1}{\sqrt{1+\omega^2 C^2 R^2}}$$

to find total noise power: $\boxed{\int_0^\infty |V_{no}|^2 df = \frac{kT}{C} = V_{no}^2}$

total noise power is independent of R

Noise Equivalent Bandwidth

An amplifier or filter has a nonideal frequency response. Noise power transmitted through is determined by the bandwidth.



Noise power $\propto$ V^2 (mean square voltage) – white noise

$$\bar{v}_i^{\,2}|A(f)|^2 = \bar{v}_o^{\,2} \ / \ Hz \text{ in a } 1Hz \text{ interval}$$

Summation over entire frequency band

$$\int_o^\infty \bar{v}_o^{\,2}(f)df = \bar{v}_i^{\,2}\int_o^\infty |A(f)|^2 df$$

We choose an equivalent BW, B, with rectangular profile whose area is the same.

$$A_m^{\,2}B = \int_o^\infty |A(f)|^2 df$$

$$B = \frac{1}{A_m^{\,2}}\int_o^\infty |A(f)|^2 df$$

This is the definition of bandwidth that we will assume in subsequent calculations.

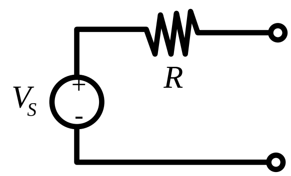
Signal-to-noise ratio

Several definitions

$$SNR = \frac{P_S}{P_N} = \frac{S}{N}$$

generally use available power

$$P_{av} = \frac{V_S^{\,2}}{4R}$$ ← *rms* voltage V_S



$\frac{S+N}{N}$ and $\frac{S+N+D}{N}$ or *SINAD* are alternate definitions.

Why is S/N important?

Affects the error rate when receiving information.

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Ref: S. Haykin, Communication Systems, 4th ed., Wiley, 2001

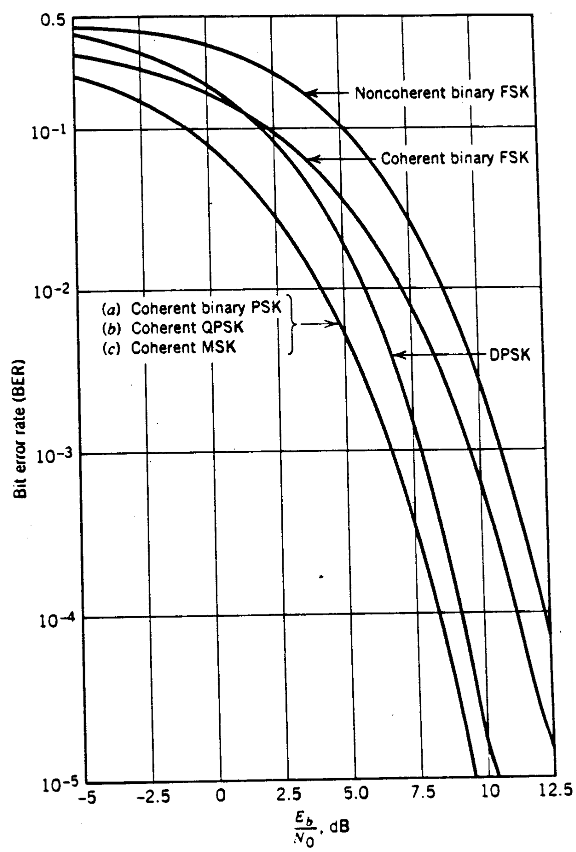
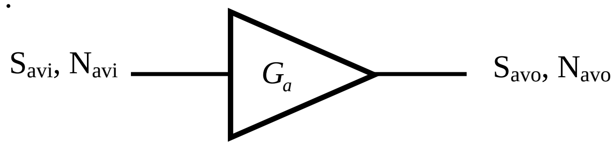


FIGURE 6.45 Comparison of the noise performance of different PSK and FSK schemes.

Noise Factor, *F*:



is a measure of how much noise is added by a component such as an amplifier.

$$F = \frac{S_{avi} / N_{avi}}{S_{avo} / N_{avo}} > 1$$

because *S/N* at input will always be greater than *S/N* at output, F > 1.

Noise factor represents the extent that S/N is degraded by the system.

$$F = \frac{\text{total noise power available at output}}{\text{noise power available at output due to source @ 290K}}$$

$$= \frac{N_{avo}}{N_{avi} \cdot G_{av}}$$

source at 290*K*

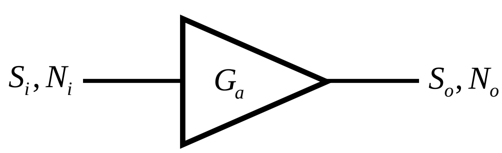
$$G_{av} = \frac{S_{avo}}{S_{avi}}$$

$$F = \frac{(S/N)_{avi}}{(S/N)_{avo}}$$

The higher the noise factor (or noise figure), the larger the degradation of S/N by the amplifier.

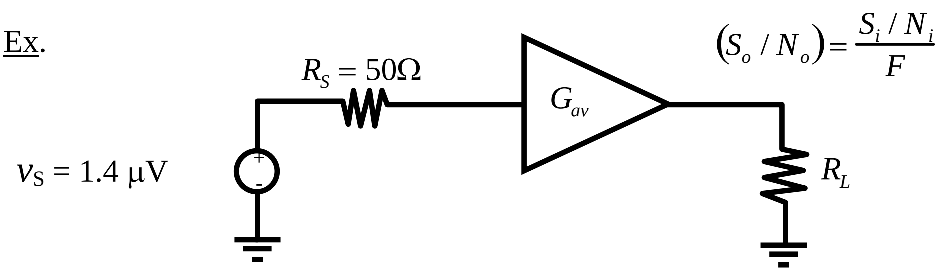
Noise Figure: $NF = 10 \log_{10} F$

Noise Factor, *F*:



is a measure of how much noise is added by a component such as an amplifier.

Ex.



$$\left.\begin{aligned} G_{av} &= 10\,dB \\ NF &= 3dB \\ B &= 10^6\,Hz \end{aligned}\right\}\ \text{amplifier specification}$$

signal available power

$$S_{avi} = \frac{v_S^2}{8R_S} = \frac{2\times10^{-12}}{400} = 5\times10^{-15}W = -113\ dBm$$

noise av. pwr. $= N_{avi} = kTB = -174 + 60 = -114dBm$

Since noise power increases with B

$10\log_{10} B = 60dB$ (in this example)

$$10\log\left(\frac{S_{avo}}{N_{avo}}\right) = 10\log\left(\frac{S_{avi}}{N_{avi}}\right) - NF$$
$$= -113 - (-114) - 3$$
$$= 1dB - 3dB = -2dB \text{ (not very good)}$$

How can S_o / N_o be improved?

1. Reduce *F*. Slight room for improvement
2. Reduce *B*. Major improvement if application can tolerate reduced *B*.
3. Increase antenna gain. Lots of room for improving Si/Ni

say $B = 10^5$

$N_{avi} = -174 + 50 = -124\, dBm$

$\frac{S_{avi}}{N_{avi}} = 11 dB$ and $\frac{S_{avo}}{N_{avo}} = 8 dB$

<u>Ex</u>. <u>Noise Floor of Spectrum Analyzer</u>

typical $NF \cong 25 dB$ for SA.

$N_{AVO} = N_{AVI} \cdot F \cdot G_{AV}$

$N_{AVI} = (-174\, dBm / Hz) + 10 \log B$ ← resolution bandwidth (RBW)

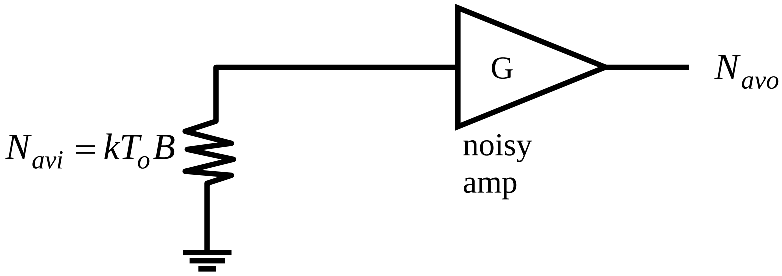
$NF = 25 dB$

$G_{AV} = 1\ (0\, dB)$

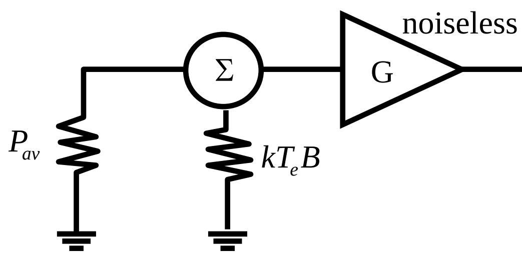
RBW	N_{AVO}
$1 kHz$	$-119\ dBm$
$10 kHz$	-109
$100 kHz$	-99
$etc.$	

We will see later how this can be improved.

The excess noise added by an active circuit such as an amplifier can also be modeled by an extra resistor at an effective input noise temperature, T_e.



is equivalent to:



$$N_{avo} = k\left(T_o + T_e\right)B \cdot G$$

(useful when $T_o \neq 290k$)

In terms of noise factor:

$$F = \frac{\text{noise out due to DUT + noise out due to source}}{\text{Noise out due to source}}$$

$$= \frac{kT_eBG + kT_oBG}{kT_oBG} = 1 + \frac{T_e}{T_o}$$

or $T_e = 290\left(F - 1\right)$

(where F is a number, not dB)

<u>Significance of T_e</u>: excess noise.

$$N_{avo}(total) = kBG\,(T_o + T_e)$$

T_o: due to source resistor

T_e: due to amplifier

<u>Example</u>: $NF = 1dB \Rightarrow F = 1.26$

$= 1 + T_e/T_o = 1 + T_e/290$

so $T_e = 75K$

total output noise $\Rightarrow 290 + 75 = 365K$ equivalent source temp

So what? Not major increase in noise power. Further reduction in F may not be justified.

But, for space application: $T_o = 20K$ is possible.

Then $T = T_o + T_e = 20 + 75 = 95K$

major degradation in noise temp.

F or NF at room temperature doesn't reveal this so clearly.

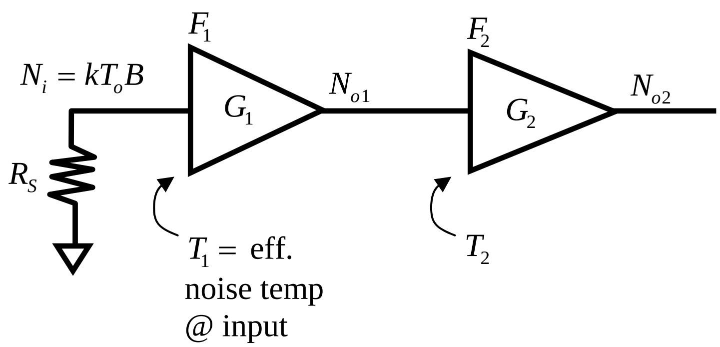
F = 1 + 75/20 = 4.5 (NF = 7 dB)

Noise Figure of Cascaded Stages. Use Available gain.

Why available gain?

Noise power defined as available power. Cascading of noise is more convenient when G_A is used.

Second Stage Noise Contribution



$$N_{o1} = k(T_o + T_1)BG_1$$

$$N_{o2} = k(T_o + T_1)BG_1G_2 + kT_2BG_2$$

To get total input referred noise power:

$$\frac{N_{o2}}{G_1G_2} = N_i \text{ (equivalent)} = k(T_o + T_1)B + kT_2B / G_1$$

excess noise at input:

$$kT_1B + kT_2B / G1$$

Recall that $F = 1 + \frac{T_e}{T_o}$

$$T_e = T_1 + T_2/G_1$$

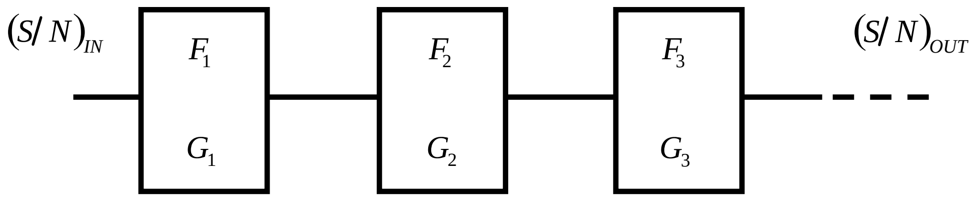
$$F_{TOTAL} = \underbrace{1 + \frac{T_1}{T_o}}_{F_1} + \frac{T_2}{\underbrace{T_o}G_1}$$

$$F_1 + \frac{F_2 - 1}{G_1}$$

Third Stage:

$$+\frac{F_3 - 1}{G_1G_2}$$

Noise Figure of Cascaded Stages



F_i = Noise Factor
G_i = Available Gain } not in dB

$$F_{TOTAL} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots$$

= Input Total Noise Factor

$$\frac{(S/N)_{IN}}{(S/N)_{OUT}} = F_{TOTAL}$$

Or: $(S/N)_{OUT}\, dB = (S/N)_{IN}\, dB - NF_{TOTAL}$

Additional stages in the cascade treated the same way.

Total available gain of cascade = $G_{a1}\, G_{a2}\, G_{a3}$...

1. <u>If noise figure is important in a receiver</u>, it is standard procedure to design so that the first stage sets the noise performance.

$$F_{TOTAL} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

This will require a large enough G_1 to diminish the noise contribution of the second stage.

2. How is the <u>minimum detectable signal</u> or MDS defined?

* at a given B (very important)

$$\underline{P_{MDS}} \Rightarrow \frac{S+N}{N} = 3dB \text{ or } S = N$$

$$\frac{S}{N} = O\ dB$$

$$P_{MDS} = 10\log(kTB) + NF\left(dB\right)$$

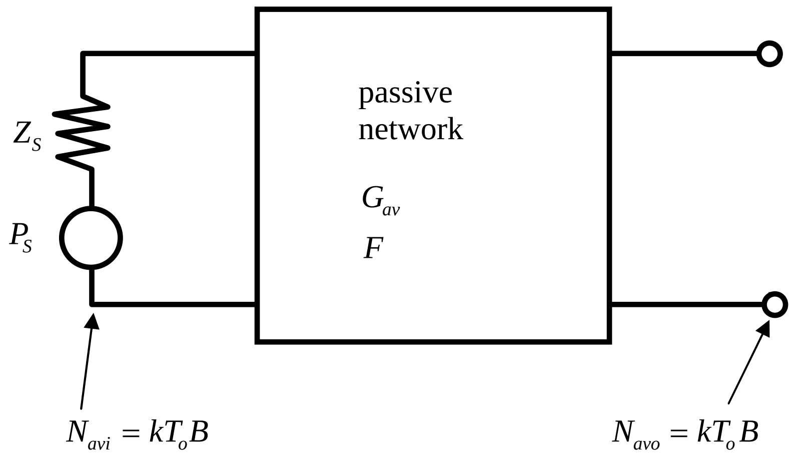
OR

$$P_{MDS} = -174\, dBm/Hz + 10\log B + NF\,(dB)$$

Noise figure of Passive Networks

ex. attenuator
filter
matching network

No active components. Only resistors and reactances.



no excess noise is generated by network

$$\frac{S_{avo}}{S_{avi}} = G_{av}$$

so, $(S/N)_i = \dfrac{P_S}{kT_oB}$

$$(S/N)_o = \frac{G \cdot P_S}{kT_oB}$$

$$F = \frac{(S/N)_i}{(S/N)_o} = \frac{1}{G}$$ Noise factor is just the inverse of gain.

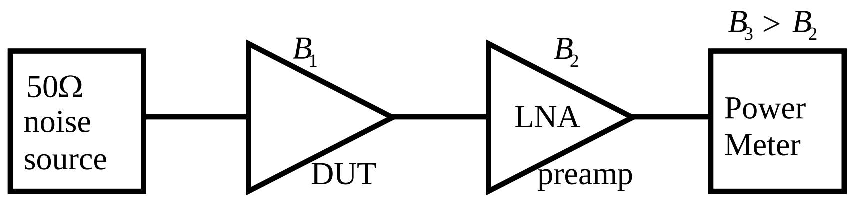
or, $NF = -G(dB)$

ex. $10dB$ attenuator $G_{av} = -10dB$

$$NF = 10\,dB$$

<u>Measuring NF: HOT-COLD NF Technique</u>

You can use a calibrated noise source for measuring NF.



$$B_n >> B_1 < B_2$$

The advantage here is that we don't need to know noise equivalent BW accurately.

Noise source has <u>very wide *BW*</u> compared with system under test.

$P_H =$ noise power with source on $= kT_H B$

$T_H =$ effective noise temp. of source

$P_o = kT_o B =$ noise power with source off.

$T_o = 290k$

<u>Excess Noise Ratio</u> $= ENR = \dfrac{P_H - P_o}{P_o} = \dfrac{T_H}{T_o} - 1$

$$ENR(dB) = 10\log_{10}\left(\frac{T_H}{T_o} - 1\right)$$

<u>*Y* factor for noise source</u>:

$$Y_S = \frac{P_H}{P_o} = \frac{T_H}{T_o}$$

So, we can use the noise source instead of the signal generator.

1. Source off. Noise power at meter:

$$P_1 = F\,kT_oB\,A_T$$

total noise factor (F) transducer gain (A_T)

2. Source on.

$$P_2 = P_1 + Y_S kT_0 B A_T$$

Divide: $$\frac{P_2}{P_1} = Y = 1 + \frac{Y_S}{F}$$

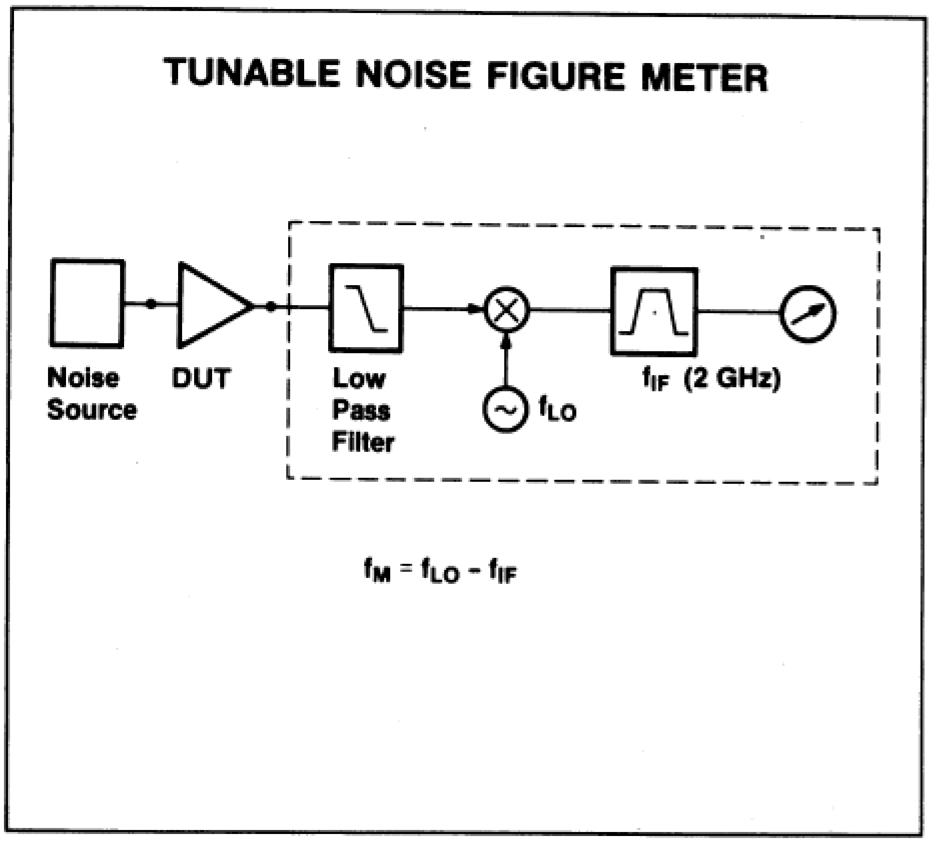
again, the transducer gain cancels, and now B cancels too. We can solve for *F* from the measured P_2/P_1.

$$F = \frac{Y_S}{Y-1}$$ Noise factor – numerical ratios, not *dB*.

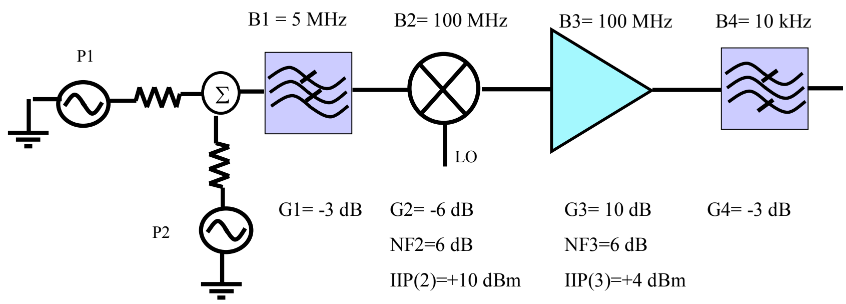
and

$$\underline{NF = 10\log F\ (dB)}$$

Block diagram of a noise figure measurement system



Noise and distortion example



Assume source P2 is off. What is the minimum source power P1 in dBm that will produce an output signal to noise ratio = 1?

First calculate noise figure:

Ftotal = F1 + (F2-1)/G1 + (F3-1)/(G1G2) + (F4-1)/(G1G2G3)

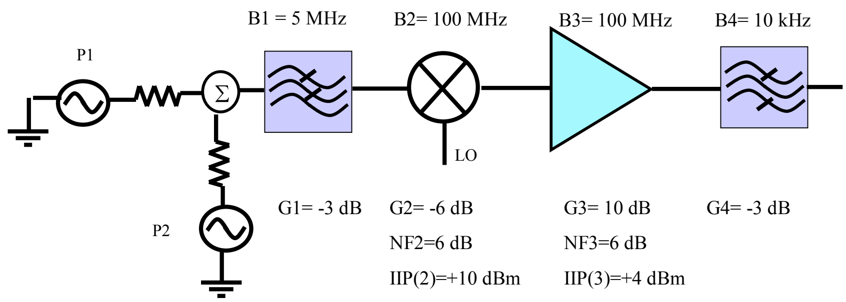
= 2 + 6 + 24 + 0.8 = 32.8 (15.1 dB NF)

Minimum signal level at input to produce (S/N)out = 1? First find the minimum bandwidth in the chain: stage 4; B4 = 10 kHz

P1 = MDS = -174 dBm/Hz + 10 log B4 + NF = -119 dBm

Could we improve the noise figure? The 3rd stage is the major contributor. We do not need such a wide band IF amplifier for a 10 KHz bandwidth, so this stage could be redesigned for minimum noise figure. Even then, the total NF is high due to the losses in stages 1 and 2. The first stage filter should be replaced with one with lower loss, since its noise figure adds directly to the receiver total NF. The best way to improve NF is to add an LNA, but this will have an impact on the IIP3.

Noise and distortion example



Now assume both sources are on and P1 = P2. How much source power will be required to produce a third order intermodulation component of - 100 dBm at the output?

First, we must calculate the input third-order intercept for the chain.

Refer the intercepts of stages 2 and 3 to the input of the filter at stage 1:

IIP(2)' = IIP(2) + 3 dB = + 13 dBm (20 mW)

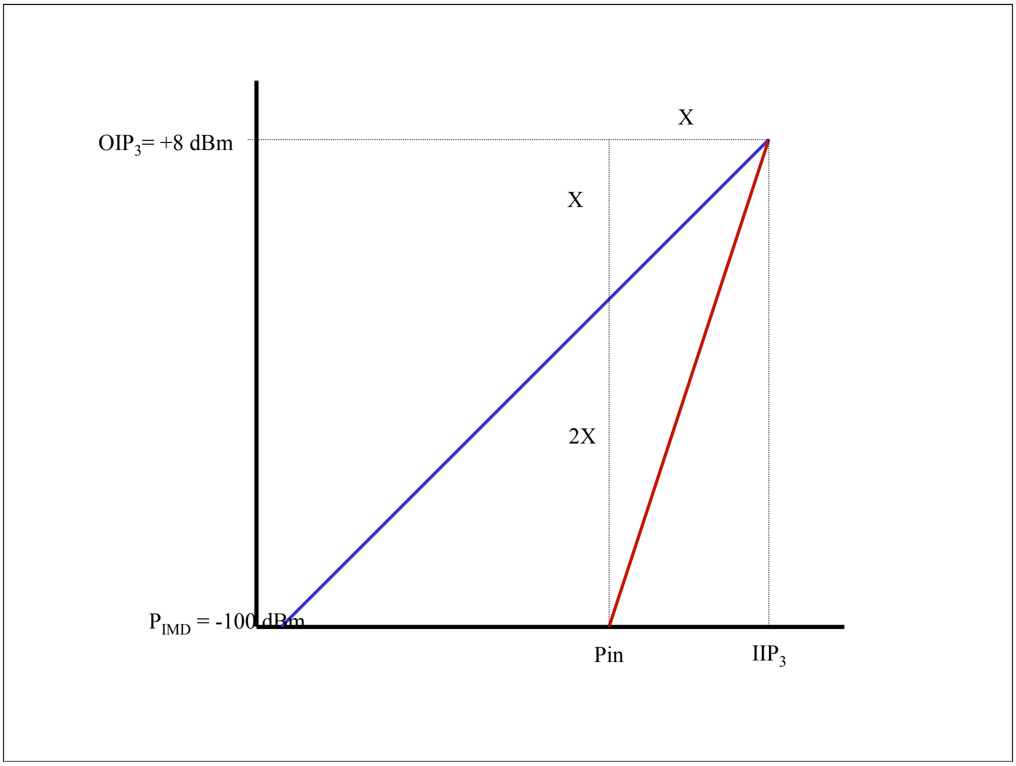
IIP(3)' = IIP(3) + 6 + 3 = +13 dBm

$(IIP_3total)^{-1}$ = 1/20 + 1/20 = 1/10 IIP_3total = +10 dBm

Next, refer to the output. Total gain of the 4 stages = -2 dB

$OIP_3total = IIP_3total - 2$ dB = +8 dBm

Next we can plot the Pout vs Pin and easily calculate Pin required for the –100 dBm IMD power.



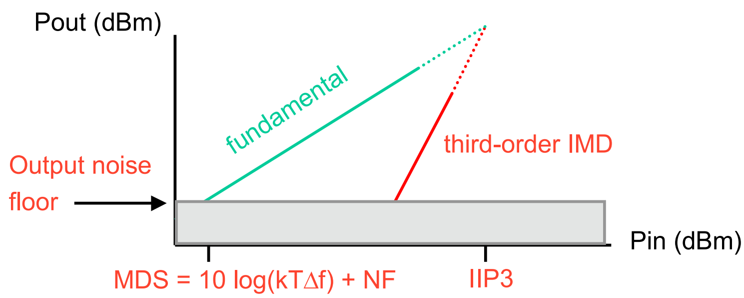
From the plot, you can see that 3x = 108 dB. Thus, x = 36 dB

Pin = IIP_3 – x = -26 dBm

Now we can calculate SFDR

Spurious Free Dynamic Range

$$SFDR = \frac{2}{3}\left[IIP3 - \left(10\log kT\Delta f + NF\right)\right]$$



IIP3 = +10 dBm MDS = -119 dBm SFDR = 86 dB

Spurious free dynamic range measures the ability of a receiver system to operate between noise limits and interference limits.

SFDR = 2 (IIP3 – MDS)/3

The maximum signal power is limited by distortion, which we describe by IIP3. The *spurious-free dynamic range* (SFDR) is a commonly used figure of merit to describe the dynamic range of an RF system. If the signal power is increased beyond the point where the IMD rises above the noise floor, then the signal-to-distortion ratio dominates and degrades by 3 dB for every 1 dB increase in signal power. If we are concerned with the third-order distortion, the SFDR is calculated from the geometric 2/3 relationship between the input intercept and the IMD.

It is important to note that the SFDR depends directly on the bandwidth Δf. It has no meaning without specifying bandwidth.

Also, we can define another receiver figure of merit: *Receiver Factor*

RxF = IIP3 – NF = 10 dBm – 15.1 dB = -5.1 dBm

The receiver factor also takes into account both noise and intermodulation properties of the system. It is independent of bandwidth.